LINEAR ALGEBRA HOMEWORK

JULY 20, 2023

Exercise 1. *Define the multiplication on $\mathbb{R}$. Hint: split a real number into two parts, greater and lesser than 0.*

Exercise 2. *Define the negation map $a \mapsto -a$ on $\mathbb{R}$.*

Exercise 3. *Denote the map*

$$[\,\cdot\,] : \mathbb{Q} \longrightarrow \mathbb{R}$$
$$a \longmapsto [a].$$

(1) *Show that $[\,\cdot\,]$ is injective.*
(2) *Show that*

$$[a + b] = [a] + [b], \ [a \times b] = [a] \times [b]$$

for any $a, b \in \mathbb{Q}$, and

$$1/[a] = [1/a]$$

for all $a \in \mathbb{Q}^\times$.

Exercise 4 (Bonus)**.** *Prove that $\mathbb{R}$ satisfies the algebra laws (N1) to (N9).*